Modified kinetic theory of Bose systems taking into account slow hydrodynamical processes

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Abstract

An approach to the description of kinetics which taking into account the large-scale hydrodynamic transport processes for quantum Bose system is proposed. The nonequilibrium statistical operator which consistently describes both the kinetic and nonlinear hydrodynamic fluctuations in quantum liquid is calculated. Using this operator the coupled equations for quantum one-particle distribution function and functional of hydrodynamic variables: densities of momentum, energies and number of particles are obtained.

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I. INTRODUCTION

The examination of dynamic properties of quantum liquids as well as of the features of transition processes from gaseous state to fluid and superfluid one with temperature decrease remains a hard problem in modern physics. The development of the nonequilibrium statistical theory which would take into account one-particle and collective physical processes that occur in a system is an example of such problem. The problem the going out beyond the hydrodynamic area to the area of an intermediate values of wave vector and frequency, where the kinetic and hydrodynamic processes are interdependent and should be considered simultaneously, is essential. The separation of contributions from the kinetic and hydrodynamic fluctuations into time correlation functions, excitations spectrum, transport coefficients allows one to obtain much more information on physical processes with the different time and spatial intervals, which define the dynamic properties of the system.

The quantum system of Bose particles serves as a physical model in theoretical descriptions both the equilibrium and nonequilibrium properties of real helium. In particular, many articles [1–22] are devoted to the hydrodynamic description of normal and superfluid states of such system. A brief review of the results of the investigations within the framework of linear hydrodynamics has been considered in the article by Tserkovnikov [20]. In papers [23–25] theoretical approaches are proposed to the description of nonlinear hydrodynamic fluctuations connected with problem of calculating the dispersion for the kinetic transport coefficients and the spectrum of collective modes in the low-frequency area for superfluid Bose liquid. The generalized Fokker-Planck equation for the nonequilibrium distribution function of slow variables for quantum systems was obtained in paper by Morozov [26]. Problems of building the kinetic equation for Bose systems based on the microscopic approach were considered in papers [26, 28]. For normal Bose systems, the calculations of the collective mode spectrum (without accounting for a thermal mode), dynamic structure factor, kinetic transport coefficients [13, see the references] are carried out on the basis of the hydrodynamic or kinetic approaches. Nevertheless, these results are valid only in the hydrodynamic area (small values of wave vector $\mathbf{k}$ and frequency ω). The papers [28–30] were devoted to the investigation of the dynamic structure factor and of collective excitations spectrum for superfluid helium.

In papers [31, 32], a generalized scheme for the theoretical description of dynamic prop-

erties of semiquantum helium has been proposed based on the method of nonequilibrium statistical operator. Here the set of equations of the generalized hydrodynamics is obtained and the thermal viscous model with kinetic and hydrodynamical collective modes is analyzed in details. The closed system of the equations for time correlation functions is obtained within the Markovian approximation for transport kernels. Using these equations the analysis of dynamic properties of semiquantum helium is carried out at two values of temperature above transition to a superfluid state. Similar investigations were performed in papers [34–36] for helium above a point of the phase transition.

In general, a hard problem exists in the description of Bose systems going out from the hydrodynamic area to the area of intermediate values of $\mathbf{k}$ and ω, where the kinetic and hydrodynamic processes are interdependent and should be considered simultaneously. This is one of the urgent problems of the statistical theory of nonequilibrium transport processes in quantum liquid. It should be noted that in the paper by Tserkovnikov [36], a problem of building of the linearized kinetic equation for the Bose system above critical temperature was considered by means of the method of two-time Green functions [37, 38].

The investigations of semiquantum helium [31, 32] became a main step in this direction. A considerable success was achieved in papers [40–42] in which the approach of the consistent description of kinetics and hydrodynamics of classical dense gases and fluids is proposed based on the method of Zubarev nonequilibrium statistical Zubarev operator [23, 44, 45, 47]. By means of this formalism the nonequilibrium statistical operator of many-particles Bose system, which consistently describes kinetics and hydrodynamics, is obtained in papers [48, 49]. The quantum nonequilibrium one-particle distribution function and the average value of density of interaction potential energy have been selected as parameters of consistent description of a nonequilibrium state.

On the other hand, the large-scale fluctuations in a system related to the slow hydrodynamical processes play the essential role at the phase transition with temperature decrease. The construction of the kinetic equations taking into account the slow processes is the hard problem for the transport theory both in classical and quantum liquids. The same problem arises at the description of low-frequency anomalies in kinetic equations related to "long tails" of correlation functions [50–54] as well as at the consistent description of collective effects in plasma [55].

The aim of the present paper is construction of the kinetic equations for quantum sys-

tem taking into account the nonlinear hydrodynamic processes using the nonequilibrium statistical operator method.

In the second part of the paper we shall obtain the nonequilibrium statistical operator and generalized transport equations for quantum Bose system, when the nonequilibrium single-particle Wigner distribution function and nonequilibrium average operator of potential interaction energy are chosen as basic parameters of the abbreviated description.

In the third part the approach to the description of kinetics taking into account the slow hydrodynamical transport processes for quantum Bose system is considered. The nonequilibrium statistical operator which consistently describes both the kinetic and nonlinear hydrodynamical fluctuations in a quantum liquid is calculated. The coupled set of kinetic equations for quantum one-particle distribution function and generalized Fokker-Plank equations for the functional of hydrodynamical variables: particles number, momentum and energy densities is obtained. Neglecting the hydrodynamic fluctuations, we obtain the traditional scheme of kinetic theory.

II. THE NONEQUILIBRIUM STATISTICAL OPERATOR OF THE CONSISTENT DESCRIPTION OF KINETICS AND HYDRODYNAMICS OF QUANTUM BOSE SYSTEM

Observable average values of energy density $\langle\hat{\varepsilon}_{\mathbf{q}}\rangle^t$, momentum density $\langle\hat{P}_{\mathbf{q}}\rangle^t$, and particle numbers density $\langle\hat{n}_{\mathbf{q}}\rangle^t$ are the abbreviated description parameters at investigations of the hydrodynamical nonequilibrium state of the normal Bose liquid which is characterized by processes of the energy, momentum and masses flows. Operators for these physical quantities are defined through the Klimontovich operator of the phase particle number density $\hat{n}_{\mathbf{q}}(\mathbf{p}) = \hat{a}^{+}_{\mathbf{p}-\frac{\mathbf{q}}{2}}\hat{a}_{\mathbf{p}+\frac{\mathbf{q}}{2}}$:

$$\hat{n}_{\mathbf{q}} = \frac{1}{\sqrt{N}}\sum_{\mathbf{p}}\hat{n}_{\mathbf{q}}(\mathbf{p}), \tag{1}$$

$$\hat{\mathbf{P}}_{\mathbf{q}} = \frac{1}{\sqrt{N}}\sum_{\mathbf{p}}\mathbf{p}\,\hat{n}_{\mathbf{q}}(\mathbf{p}), \tag{2}$$

$$\hat{\varepsilon}^{kin}_{\mathbf{q}} = \frac{1}{\sqrt{N}}\sum_{\mathbf{p}}\left(\frac{p^2}{2m} - \frac{q^2}{8m}\right)\hat{n}_{\mathbf{q}}(\mathbf{p}), \tag{3}$$

$$\hat{\varepsilon}^{int}_{\mathbf{q}} = \frac{1}{\sqrt{N}}\sum_{\mathbf{p}}\sum_{\mathbf{p}'}\sum_{\mathbf{k}}\nu(k)\hat{a}^{+}_{\mathbf{p}+\frac{\mathbf{k}-\mathbf{q}}{2}}\,\hat{n}_{\mathbf{q}}(\mathbf{p}')\hat{a}_{\mathbf{p}-\frac{\mathbf{k}-\mathbf{q}}{2}}, \tag{4}$$

where $\hat{\varepsilon}_{\mathbf{q}}^{kin}$ and $\hat{\varepsilon}_{\mathbf{q}}^{int}$ are Fourier-components of the operators of kinetic and potential energy densities. Average value of the phase particles number density operator is equal to the nonequilibrium one-particle distribution function $f_1(\mathbf{q},\mathbf{p},t) = \langle\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t$, which satisfies the kinetic equation for quantum Bose system.

The agreement between kinetics and hydrodynamics for dilute Bose gas does not cause problems because in this case the density is a small parameter. Therefore, only the quantum one-particle distribution function $f_1(\mathbf{q},\mathbf{p};t)$ can be chosen for parameter of the abbreviated description. At transition to quantum Bose liquids, the contribution of collective correlations, which are described by average potential energy of interaction, is more important than one-particle correlations connected with $f_1(\mathbf{q},\mathbf{p};t)$. From this fact it follows that for consistent description of kinetics and hydrodynamics of Bose liquid, the one-particle nonequilibrium distribution function along with the average potential energy of interaction are necessarily should be chosen as the parameters of the abbreviated description [48, 49]. The nonequilibrium state of such quantum system is completely described by the nonequilibrium statistical operator $\hat{\varrho}(t)$ which satisfies the quantum Liouville equation:

$$\frac{\partial}{\partial t}\hat{\varrho}(t) + i\hat{L}_N\hat{\varrho}(t) = -\varepsilon\left(\hat{\varrho}(t) - \hat{\varrho}_q(t)\right). \tag{5}$$

The infinitesimal source ε in the right-hand side of this equation breaks symmetry of Liouville equation with respect to $t \to -t$ and selects retarded solutions ($\varepsilon \to +0$ after limiting thermodynamic transition). The quasiequilibrium statistical operator $\hat{\varrho}_q(t)$ is determined from the condition of the informational entropy extremum of systems at the conservation of normalization condition $\mathrm{Sp}\,\hat{\varrho}_q(t) = 1$ for fixed values $\langle\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t$ and $\langle\hat{\varepsilon}_{\mathbf{q}}^{int}\rangle^t$ [48, 49]:

$$\hat{\varrho}_q(t) = \exp\left\{-\Phi(t) - \sum_{\mathbf{q}}\beta_{-\mathbf{q}}(t)\hat{\varepsilon}_{\mathbf{q}}^{int} - \sum_{\mathbf{q}}\sum_{\mathbf{p}}\gamma_{-\mathbf{q}}(\mathbf{p};t)\hat{n}_{\mathbf{q}}(\mathbf{p})\right\}, \tag{6}$$

where the Lagrangian multipliers $\beta_{-\mathbf{q}}(t)$, $\gamma_{-\mathbf{q}}(\mathbf{p};t)$ are determined from the self-consistent conditions:

$$\langle\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t = \langle\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle_q^t, \quad \langle\hat{\varepsilon}_{\mathbf{q}}^{int}\rangle^t = \langle\hat{\varepsilon}_{\mathbf{q}}^{int}\rangle_q^t.$$

The Massieu-Plank functional

$$\Phi(t) = \ln\,\mathrm{Sp}\,\exp\left\{-\sum_{\mathbf{q}}\beta_{-\mathbf{q}}(t)\hat{\varepsilon}_{\mathbf{q}}^{int} - \sum_{\mathbf{q}}\sum_{\mathbf{p}}\gamma_{-\mathbf{q}}(\mathbf{p};t)\hat{n}_{\mathbf{q}}(\mathbf{p})\right\} \tag{7}$$

is determined from the normalization condition. Here $\langle(...)\rangle^t = \mathrm{Sp}(...)\hat{\varrho}(t)$, $\langle(...)\rangle^t_q = \mathrm{Sp}(...)\hat{\varrho}_q(t)$. At given quasiequilibrium statistical operator $\hat{\varrho}_q(t)$ we can find the nonequilibrium statistical operator $\hat{\varrho}(t)$ that satisfies the quantum Liouville equation in the presence of a source:

$$\hat{\varrho}(t) = \hat{\varrho}_q(t) + \sum_{\mathbf{q}} \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} T_q(t,t') \int\limits_0^1 d\tau \left(\hat{\varrho}_q(t)\right)^{\tau} I^{int}_{\varepsilon}(\mathbf{q},t') \left(\hat{\varrho}_q(t)\right)^{1-\tau} \beta_{-\mathbf{q}}(t') \tag{8}$$

$$+ \sum_{\mathbf{q}} \sum_{\mathbf{p}} \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} T_q(t,t') \times \int\limits_0^1 d\tau \left(\hat{\varrho}_q(t)\right)^{\tau} I_n(\mathbf{p},\mathbf{q},t') \left(\hat{\varrho}_q(t)\right)^{1-\tau} \gamma_{-\mathbf{q}}(\mathbf{p},t'),$$

where the generalized flows

$$I^{int}_{\varepsilon}(\mathbf{q},t) = \Big(1 - P(t)\Big) i\hat{L}_N \hat{\varepsilon}^{int}_{\mathbf{q}}, \tag{9}$$

$$I_n(\mathbf{p},\mathbf{q},t) = \Big(1 - P(t)\Big) i\hat{L}_N \hat{n}_{\mathbf{q}}(\mathbf{p}) \tag{10}$$

contain generalized Mori projection operator. $T_q(t,t')$ is the generalized evolution Kawasaki-Gunton operator with the projection [48, 49]. The nonequilibrium statistical operator (8) is obtained at abbreviated description of kinetics and hydrodynamics of Bose system. Using it, we can find the non-closed system of transport equations for the parameters of abbreviated description $\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t$ and $\langle\hat{\varepsilon}^{int}_{\mathbf{q}}\rangle^t$ [5, 6]:

$$\frac{\partial}{\partial t}\langle\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t = \langle\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p})\rangle^t_q + \sum_{\mathbf{q}'} \int\limits_{-\infty}^{t} dt'\, \mathrm{e}^{\epsilon(t'-t)} \varphi^{int}_{n\varepsilon}(\mathbf{q},\mathbf{p},\mathbf{q}',t,t')\beta_{-\mathbf{q}}(t') \tag{11}$$

$$+ \sum_{\mathbf{q}'} \sum_{\mathbf{p}'} \int\limits_{-\infty}^{t} dt'\, \mathrm{e}^{\epsilon(t'-t)}\, \varphi_{nn}(\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{q}',t,t')\gamma_{-\mathbf{q}}(\mathbf{p}',\mathbf{t}'),$$

$$\frac{\partial}{\partial t}\langle\hat{\varepsilon}^{int}_{\mathbf{q}}\rangle^t = \langle\dot{\hat{\varepsilon}}^{int}_{\mathbf{q}}\rangle^t_q + \sum_{\mathbf{q}'} \int\limits_{-\infty}^{t} dt'\, \mathrm{e}\epsilon(t'-t) \varphi^{int}_{\varepsilon\varepsilon}(\mathbf{q},\mathbf{q}',t,t')\beta_{-\mathbf{q}}(t')$$

$$+ \sum_{\mathbf{q}'} \sum_{\mathbf{p}'} \int\limits_{-\infty}^{t} dt'\, \mathrm{e}^{\epsilon(t'-t)}\, \varphi^{int,int}_{\varepsilon n}(\mathbf{q},\mathbf{q}',\mathbf{p}',t,t')\gamma_{-\mathbf{q}}(\mathbf{p}',t').$$

In the equations (11) the generalized transport kernels, which describe dissipative processes in the system, are introduced as follows:

$$\varphi^{int}_{n\varepsilon}(\mathbf{q},\mathbf{p},\mathbf{q}',t,t') = \mathrm{Sp}\left[I_n(\mathbf{q},p,t)T_q(t,t') \int\limits_0^1 d\tau \varrho^{\tau}_q(t') I^{int}_{\varepsilon}(\mathbf{q}',t') \varrho^{1-\tau}_q(t')\right], \tag{12}$$

$$\varphi^{int}_{\varepsilon n}(\mathbf{q},\mathbf{q}',\mathbf{p},t,t') = \mathrm{Sp}\left[I^{int}_{\varepsilon}(\mathbf{q},t')T_q(t,t')\int\limits_0^1 d\tau\,\varrho^{\tau}_q(t')I_n(\mathbf{p},\mathbf{q}',t)\varrho^{1-\tau}_q(t')\right], \tag{13}$$

$$\varphi_{nn}(\mathbf{q},\mathbf{p},\mathbf{q}',\mathbf{p}',t,t') = \mathrm{Sp}\left[I_n(\mathbf{q},\mathbf{p},t)T_q(t,t')\int\limits_0^1 d\tau\,\varrho^{\tau}_q(t')I_n(\mathbf{p}',\mathbf{q}',t)\varrho^{1-\tau}_q(t')\right], \tag{14}$$

$$\varphi^{int}_{\varepsilon\varepsilon}(\mathbf{q},\mathbf{q}',t,t') = \mathrm{Sp}\left[I^{int}_{\varepsilon}(\mathbf{q},t')T_q(t,t')\int\limits_0^1 d\tau\,\varrho^{\tau}_q(t')I^{int,int}_{\varepsilon}(\mathbf{q}',t')\varrho^{1-\tau}_q(t')\right]. \tag{15}$$

The system of equations (11) for the one-particle distribution function and the average density of potential energy is strongly nonlinear and it can be used to description both the strongly and weakly nonequilibrium states of the Bose system with a consistent consideration of kinetics and hydrodynamics. The description of weakly nonequilibrium processes was reviewed in [49]. Projecting transport equations on the values of the component of the vector $\mathbf{\Psi}(\mathbf{p}) = \left(1,\, \mathbf{p},\, \frac{p^2}{2m} - \frac{q^2}{8m}\right)$, we shall obtain the equations of nonlinear hydrodynamics, in which the transport processes of kinetic and potential parts of energy are described by two interdependent equations. Obviously, such equations of the nonlinear hydrodynamic processes give more opportunity to describe the process of mutual transformation of kinetic and potential energy in detail at investigation of nonequilibrium processes occurring in the system.

The proposed scheme of transport equations is inconvenient when considering the kinetics and hydrodynamics in the vicinity of the phase transition point, where the large-scale fluctuations play the essential role.

III. KINETIC EQUATION FOR THE NONEQUILIBRIUM WIGNER FUNCTION AND FOKKER-PLANCK EQUATION FOR DISTRIBUTION FUNCTION OF THE HYDRODYNAMIC VARIABLES

As previously, the nonequilibrium quantum distribution function $f_1(\mathbf{q},\mathbf{p};t) = \langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t$ is chosen as parameter to description of one-particle correlations. We introduce the distribution function of hydrodynamic variables to description of the collective processes in a quantum system as follows:

$$\hat{f}(a) = \frac{1}{(2\pi)^5}\int d\mathbf{x}\, \mathrm{e}^{i\mathbf{x}(\hat{\mathbf{a}}-\mathbf{a})}, \tag{16}$$

where $\hat{\mathbf{a}} = \{\hat{a}_{1\mathbf{k}}, \hat{a}_{2\mathbf{k}}, \hat{a}_{3\mathbf{k}}\}$, $\hat{a}_{1\mathbf{k}} = \hat{n}_{\mathbf{k}}$, $\hat{a}_{2\mathbf{k}} = \hat{\mathbf{P}}_{\mathbf{k}}$, $\hat{a}_{3\mathbf{k}} = \hat{\varepsilon}_{\mathbf{k}} = \hat{\varepsilon}^{kin}_{\mathbf{k}} + \hat{\varepsilon}^{int}_{\mathbf{k}}$ are the Fourier-components of the operators of particles number, momentum and energy densities (1)–(4). The scalar values $a_{m\mathbf{k}} = \{n_{\mathbf{k}},\ \mathbf{P}_{\mathbf{k}},\ \varepsilon_{\mathbf{k}}\}$ are the corresponding collective variables. The operator function (16) is obtained in accordance with Weyl correspondence rule from the classical distribution function [26]

$$f(a) = \delta(\mathbf{A} - \mathbf{a}) = \prod_{m=1}^{N} \prod_{\mathbf{k}} \delta(A_{m\mathbf{k}} - a_{m\mathbf{k}}),$$

where $\mathbf{A} = \{A_{1\mathbf{k}} \ldots, A_{N\mathbf{k}}\}$ is the classical dynamical variables.

The average values $f_1(\mathbf{q}, \mathbf{p}; t) = \langle \hat{n}_{\mathbf{k}}(\mathbf{p})\rangle^t$, $f(a;t) = \langle \hat{f}(a)\rangle^t$ are calculated using the nonequilibrium statistical operator $\hat{\varrho}(t)$, which satisfy the Liouville equation. In line with the idea of abbreviated description of the nonequilibrium state, the statistical operator $\hat{\varrho}(t)$ must functionally depend on the quantum one-particle distribution function and distribution functions of the hydrodynamic variables:

$$\hat{\varrho}(t) = \hat{\varrho}\left(\ldots\ f_1(\mathbf{q}, \mathbf{p}; t),\ f(a;t)\ \ldots\right). \tag{17}$$

Thus, the task is to find the solution of the Liouville equation for $\hat{\varrho}(t)$ which has the form (17). For that purpose we use the method of Zubarev nonequilibrium statistical operator [23, 44, 45, 47]. We consider the Liouville equation (5) with infinitely small source. The source correctly selects retarded solutions in accordance with the abbreviated description of nonequilibrium state of a system. The quasiequilibrium statistical operator $\hat{\varrho}_q(t)$ is determined in the usual way, from the condition of the maximum informational entropy functional with the normalization condition: $\mathrm{Sp}\,\hat{\varrho}_q(t) = 1$. Then the quasiequilibrium statistical operator can be written as

$$\hat{\varrho}_q(t) = \exp\left\{-\Phi(t) - \sum_{\mathbf{q}}\sum_{\mathbf{p}} \gamma_{-\mathbf{q}}(\mathbf{p};t)\hat{n}_{\mathbf{q}}(\mathbf{p}) - \int da\, F(a;t)\, \hat{f}(a)\right\}, \tag{18}$$

where $da \ \rightarrow\ \{dn_{\mathbf{k}},\ d\mathbf{P}_{\mathbf{k}},\ d\varepsilon_{\mathbf{k}}\}$.

The Massieu-Plank functional $\Phi(t)$ is determined from the normalization condition:

$$\Phi(t) = \ln\,\mathrm{Sp}\,\left[\exp\left\{-\sum_{\mathbf{qp}} \gamma_{-\mathbf{q}}(\mathbf{p};t)\hat{n}_{\mathbf{q}}(\mathbf{p}) - \int da\, F(a;t)\, \hat{f}(a)\right\}\right].$$

Functions $\gamma_{-\mathbf{q}}(\mathbf{p};t)$ and $F(a,t)$ are the Lagrange multipliers and can be defined from the self-consistent conditions:

$$f_1(\mathbf{q}, \mathbf{p}; t) = \langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t = \langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t_q, \quad f(a;t) = \langle \hat{f}(a)\rangle^t = \langle \hat{f}(a)\rangle^t_q. \tag{19}$$

It is convenient to rewrite the quasiequilibrium statistical operator (18) in the following form:

$$\hat{\varrho}_q(t) = \int da \int_0^1 d\tau (\hat{\varrho}_q^{kin}(t))^{\tau} \mathrm{e}^{-F(a;t)\hat{f}(a)} (\hat{\varrho}_q^{kin}(t))^{1-\tau}, \tag{20}$$

where

$$\hat{\varrho}_q^{kin}(t) = \exp\left\{ -\Phi(t) - \sum_{\mathbf{q}\,\mathbf{p}} \gamma_{-\mathbf{q}}(\mathbf{p};t)\hat{n}_{\mathbf{q}}(\mathbf{p}) \right\} \tag{21}$$

is the "kinetic" quasiequilibrium statistical operator. Using the self-consistent conditions (19) we find the function $F(a;t)$:

$$\mathrm{e}^{-F(a;t)} = \int da'\, W_{-1}(a,a';t) f(a';t), \tag{22}$$

where function $W_{-1}(a,a';t)$ is determined from integral equation

$$\int da''\, W(a,a'';t)\, W_{-1}(a'',a';t) = \delta(a-a'). \tag{23}$$

$$W(a,a';t) = \mathrm{Sp}\left[\hat{f}(a) \int_0^1 d\tau (\hat{\rho}_q^{kin}(t))^{\tau} \hat{f}(a') (\hat{\rho}_q^{kin}(t))^{1-\tau} \right] \tag{24}$$

is the structural distribution function of hydrodynamic fluctuations averaged with "kinetic" quasiequilibrium operator. The functions $W(a,a';t)$ and $W_{-1}(a,a';t)$ satisfy equation (23) and contain the singular and regular terms [26]:

$$\begin{aligned} W(a,a';t) &= W(a,t)[\delta(a-a') - R(a,a';t)], \\ W_{-1}(a,a';t) &= W^{-1}(a,t)[\delta(a-a') - r(a,a';t)], \\ \int da\, W(a;t)\, R(a,a';t) &= \int da'\, R(a,a';t) = 0, \\ \int da\, W(a;t)\, r(a,a';t) &= \int da'\, r(a,a';t) = 0. \end{aligned} \tag{25}$$

This function can be consider as a Jacobian of the transition in the collective variables space $n_{\mathbf{k}}, \mathbf{P}_{\mathbf{k}}, \varepsilon_{\mathbf{k}}$, which are averaged with the "kinetic" quasiequilibrium statistical operator.

Taking into account (22), the initial quasiequilibrium operator (20) can be represented as [26]:

$$\hat{\varrho}_q(t) = \int da \int da' \int_0^1 d\tau (\hat{\varrho}_q^{kin}(t))^{\tau} \hat{f}(a) (\hat{\varrho}_q^{kin}(t))^{1-\tau}\, W_{-1}(a,a';t) f(a';t), \tag{26}$$

or

$$\hat{\varrho}_q(t) = \int da f(a;t) \hat{\varrho}_L(t), \tag{27}$$

where

$$\hat{\varrho}_L(a;t) = \int da' \int_0^1 d\tau (\hat{\varrho}_q^{kin}(t))^{\tau} \hat{f}(a';t)\,(\hat{\varrho}_q^{kin}(t))^{1-\tau}. \tag{28}$$

Gibbs entropy which corresponds to the quasiequilibrium statistical operator (26) can be written in the form:

$$S(t) = -\langle \ln \hat{\varrho}_q(t)\rangle_q^t = \Phi(t) + \sum_{\mathbf{qp}} \gamma_{-\mathbf{q}}(\mathbf{p};t)\langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle_q^t - \int da f(a;t) \ln\left(\int da' W_{-1}(a,a')f(a';t)\right), \tag{29}$$

from which, taking into account the self-consistency condition (19), we obtain nonequilibrium entropy of the Bose-system:

$$S(t) = \Phi(t) + \sum_{\mathbf{qp}} \gamma_{-\mathbf{q}}(\mathbf{p};t)\langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t - \int da f(a;t) \ln\left(\int da' W_{-1}(a,a')f(a';t)\right), \tag{30}$$

that contains the kinetic and hydrodynamic contributions.

After constructing the quasiequilibrium statistical operator (27), the Liouville equation (5) for the operator $\Delta\hat{\varrho}(t) = \hat{\varrho}(t) - \hat{\varrho}_q(t)$ is written in the form:

$$\left(\frac{\partial}{\partial t} + i\hat{L}_N + \varepsilon\right)\Delta\hat{\varrho}(t) = \left(\frac{\partial}{\partial t} + i\hat{L}_N\right)\hat{\varrho}_q(t). \tag{31}$$

Time derivative of the right-hand side of this equation can be expressed through the projection Kawasaki-Gunton operator $P_q(t)$ [23, 26, 47]:

$$\frac{\partial}{\partial t}\hat{\varrho}_q(t) = -P_q(t) i\hat{L}_N \hat{\varrho}(t). \tag{32}$$

In our case the projection operator acts arbitrary on statistical operators $\hat{\varrho}'$ according to the rule

$$P_q(t)\hat{\varrho}' = \hat{\varrho}_q(t)\mathrm{Sp}\hat{\varrho}' + \sum_{\mathbf{q}\,\mathbf{p}} \frac{\partial \hat{\varrho}_q(t)}{\partial \langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t}\left[\mathrm{Sp}\left(\hat{n}_{\mathbf{q}}(\mathbf{p})\hat{\varrho}'\right) - \langle \hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t \mathrm{Sp}\hat{\varrho}'\right] + \int da \frac{\partial \hat{\varrho}_q(t)}{\partial f(a;t)}\left[\mathrm{Sp}\left(\hat{f}(a)\hat{\varrho}'\right) - f(a;t)\mathrm{Sp}\hat{\varrho}'\right]. \tag{33}$$

Taking into account relation (32), we rewrite the equation (31) as follows:

$$\left(\frac{\partial}{\partial t} + (1 - P_q(q))\, iL_N + \varepsilon\right)\Delta\hat{\varrho}(t) = -\left(1 - P_q(q)\right) i\hat{L}_N \hat{\varrho}_q(t). \tag{34}$$

Formal solution of (34) is

$$\Delta\hat{\varrho(t)} = -\int_{-\infty}^{t} dt'\, \mathrm{e}^{\varepsilon(t'-t)} T_q(t;t')\,(1 - P_q(t))\, i\hat{L}_N \hat{\varrho}_q(t), \tag{35}$$

where

$$T_q(t;t') = \exp_+\left\{-\int\limits_{t'}^{t} dt'\,(1-P_q(t'))\,i\hat{L}_N\right\} \tag{36}$$

is the generalized time evolution operator, that take into account projection. From (35) we find the nonequilibrium statistical operator

$$\hat{\varrho}(t) = \hat{\varrho}_q(t) - \int\limits_{-\infty}^{t} dt'\,\mathrm{e}^{\varepsilon(t'-t)}T_q(t;t')\,(1-P_q(t))\,iL_N\hat{\varrho}_q(t). \tag{37}$$

Now we consider the operation of Liouville operator on the quasiequilibrium operator (26):

$$\begin{aligned} i\hat{L}_N\hat{\varrho}_q(t) = &-\sum_{\mathbf{q}\,\mathbf{p}}\gamma_{-\mathbf{q}}(\mathbf{p};t)\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p})(\hat{\varrho}_q(t))^{1-\tau} \\ &-\int daF(a;t)\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau i\hat{L}_N\hat{f}(a)(\hat{\varrho}_q(t))^{1-\tau}, \end{aligned} \tag{38}$$

where $\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p}) = i\hat{L}_N\hat{n}_{\mathbf{q}}(\mathbf{p})$. Since [26]

$$i\hat{L}_N\hat{f}(a) = -\frac{\partial}{\partial a}\hat{J}(a) \tag{39}$$

with

$$\hat{J}(a) = (2\pi)^{-N}\int dx e^{ix(\hat{a}-a)}\int_0^1 d\tau e^{-i\tau x\hat{a}}i\hat{L}_N\hat{a}e^{i\tau x\hat{a}}, \tag{40}$$

the second term on the right-hand side of (38) can be represented:

$$\begin{aligned} &\int daF(a;t)\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau(-\frac{\partial}{\partial a}\hat{J}(a))(\hat{\varrho}_q(t))^{1-\tau} = \\ &\int da(\frac{\partial}{\partial a}F(a;t))\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau\hat{J}(a)(\hat{\varrho}_q(t))^{1-\tau} \end{aligned}$$

and using (22) it can be written as follows:

$$\begin{aligned} &\int da(\frac{\partial}{\partial a}F(a;t))\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau\hat{J}(a)(\hat{\varrho}_q(t))^{1-\tau} = \\ -&\int da\left[\frac{\partial}{\partial a}\ln\int da'W_{-1}(a,a')f(a';t)\right]\int\limits_0^1 d\tau(\hat{\varrho}_q(t))^\tau\hat{J}(a)(\hat{\varrho}_q(t))^{1-\tau}. \end{aligned} \tag{41}$$

Then the expression (38) taking into account (38) will take the form:

$$i\hat{L}_N\hat{\varrho}_q(t) = -\sum_{\mathbf{q},\mathbf{p}}\gamma_{-\mathbf{q}}(\mathbf{p};t)\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p};\tau)\hat{\varrho}_q(t) + \tag{42}$$
$$\int da\hat{J}(a;\tau)\left[\frac{\partial}{\partial a}\ln\int da' W_{-1}(a,a';t)f(a';t)\right]\hat{\varrho}_q(t),$$

where

$$\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p};\tau) = \int_0^1 d\tau(\hat{\varrho}_q(t))^{\tau}\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p})(\hat{\varrho}_q(t))^{-\tau},\ \hat{J}(a,\tau) = \int_0^1 d\tau(\hat{\varrho}_q(t))^{\tau}\hat{J}(a)(\hat{\varrho}_q(t))^{-\tau}. \tag{43}$$

Now in accordance with (41) and (27) the Liouville operator action on $\hat{\varrho}_q(t)$ can be represented as:

$$i\hat{L}_N\hat{\varrho}_q(t) = -\int da\sum_{\mathbf{q},\mathbf{p}}\gamma_{-\mathbf{q}}(\mathbf{p};t)\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p},\tau)f(a;t)\hat{\varrho}_L(a;t)$$
$$+\int da\int a''\sum_{\mathbf{k}}\left[\hat{J}(n_{\mathbf{k}};\tau)\frac{\partial}{\partial n_{\mathbf{k}}} + \hat{J}(\mathbf{P}_{\mathbf{k}};\tau)\frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} + \hat{J}(\varepsilon_{\mathbf{k}};\tau)\frac{\partial}{\partial\hat{\varepsilon}_{\mathbf{k}}(\tau)}\right]$$
$$\times\ln\left[\int da' W_{-1}(a,a';t)f(a';t)\right]f(a'';t)\hat{\varrho}_L(a'';t').$$

Finally, substituting this expression in (37), we obtain the nonequilibrium statistical operator:

$$\hat{\varrho}(t) = \int da f(a;t)\hat{\varrho}_L(a,t) + \int da\sum_{\mathbf{q}\,\mathbf{p}}\int_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)}T_q(t,t') \tag{44}$$
$$\times\left(1-P_q(t')\right)\dot{\hat{n}}_{\mathbf{q}}(\mathbf{p},\tau)\hat{\varrho}_L(a;t')f(a;t)\gamma_{-\mathbf{q}}(\mathbf{p};t')$$
$$+\int da\int a''\sum_{\mathbf{k}}\int_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)}T_q(t,t')\left(1-P_q(t')\right)$$
$$\times\left[\hat{J}(n_{\mathbf{k}};\tau)\frac{\partial}{\partial n_{\mathbf{k}}} + \hat{J}(\mathbf{P}_{\mathbf{k}};\tau)\frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} + \hat{J}(\varepsilon_{\mathbf{k}};\tau)\frac{\partial}{\partial\hat{\varepsilon}_{\mathbf{k}}(\tau)}\right]$$
$$\times\ln\left[\int da' W_{-1}(a,a';t)f(a';t)\right]f(a'';t)\hat{\varrho}_L(a'';t').$$

This formula gives the nonequilibrium statistical operator that consistently describes the kinetic and nonlinear hydrodynamic fluctuations of quantum Bose fluids. After neglecting the hydrodynamic fluctuations we can return to the traditional scheme accepted in the kinetic theory. Nonequilibrium statistical operator is the functional of the abbreviated description parameters $f_1(\mathbf{q},\mathbf{p};\mathbf{t})$, $f(a;t)$, which are required for complete description of the

transport equations. For this we use relations:

$$\frac{\partial}{\partial t} f_1(\mathbf{q},\mathbf{p};t) = \langle \dot{\hat{n}}_{\mathbf{q}}(\mathbf{p})\rangle^t = \langle \dot{\hat{n}}_{\mathbf{q}}(\mathbf{p})\rangle^t_q + \langle I_n(\mathbf{q},\mathbf{p})\rangle^t,$$

$$\frac{\partial}{\partial t} f(a;t) = \mathrm{Sp}\left\{\hat{\varrho}_L(a,t) i\hat{L}_N \hat{f}(a)\right\}, \tag{45}$$

where $I_n(\mathbf{q},\mathbf{p};t)$ is the generalized flow of density. Thus, substitution of explicit expression (44) into these relations and after simple but somewhat unwieldy transformations, we obtain the final expressions for kinetic equations:

$$\left(\frac{\partial}{\partial t} - i\frac{\mathbf{qp}}{m}\right) f_1(\mathbf{q},\mathbf{p};t) - \int da f(a;t')\langle i\hat{L}_N^{int}\hat{n}_{\mathbf{q}}(\mathbf{p})\rangle^t_L \tag{46}$$

$$= \sum_{\mathbf{q}'\,\mathbf{p}'} \int da \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} \Phi_{nn}(\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{p}';t,t') f(a;t')\gamma_{\mathbf{q}'}(\mathbf{p}';t)$$

$$-\sum_{\mathbf{q}'\,\mathbf{p}'}\sum_{\mathbf{k}} \int da \int da'' \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} \left\{ \Phi_{n\mathbf{P}}(\mathbf{q},\mathbf{p},a,a'';t,t')\frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}}\right.$$

$$\left.+\Phi_{n\varepsilon}(\mathbf{q},\mathbf{p},a,a'';t,t')\frac{\partial}{\partial \varepsilon_{\mathbf{k}}}\right\} \left[\ln \int da' W_{-1}(a,a';t) f(a,';t)\right] f(a'';t),$$

$$\frac{\partial}{\partial t} f(a;t) + \sum_{\mathbf{k}} \left\{ \frac{\partial}{\partial n_{\mathbf{k}}} \int da' v_n(a,a';t) f(a';t)\right.$$

$$\left. + \frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} \int da' v_{\mathbf{P}}(a,a';t) f(a';t) + \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \int da' v_{\varepsilon}(a,a';t) f(a';t) \right\} \tag{47}$$

$$= \sum_{\mathbf{k}} \frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} \sum_{\mathbf{q}'\,\mathbf{p}'} \int da' \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} \times \Phi_{\mathbf{P}n}(a,a'',\mathbf{q}',\mathbf{p}';t,t') f(a';t')\gamma_{\mathbf{q}'}(\mathbf{p}';t')$$

$$+ \sum_{\mathbf{k}} \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \sum_{\mathbf{q}'\,\mathbf{p}'} \int da' \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} \Phi_{\varepsilon n}(a,a',\mathbf{q}',\mathbf{p}';t,t') f(a';t')\gamma_{\mathbf{q}'}(\mathbf{p}';t')$$

$$- \sum_{\mathbf{k}\,\mathbf{q}'} \int da'' \int\limits_{-\infty}^{t} dt' \mathrm{e}^{\varepsilon(t'-t)} \left[\frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} \Phi_{\mathbf{PP}}(a,a',a'';t,t') \frac{\partial}{\partial \mathbf{P}_{\mathbf{q}}}\right.$$

$$+ \frac{\partial}{\partial \mathbf{P}_{\mathbf{k}}} \Phi_{\mathbf{P}\varepsilon}(a,a',a'';t,t') \frac{\partial}{\partial \varepsilon_{\mathbf{q}}} + \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \Phi_{\varepsilon\mathbf{P}}(a,a',a'';t,t') \frac{\partial}{\partial \mathbf{P}_{\mathbf{q}}}$$

$$\left. + \frac{\partial}{\partial \varepsilon_{\mathbf{k}}} \Phi_{\varepsilon\varepsilon}(a,a',a'';t,t') \frac{\partial}{\partial \varepsilon_{\mathbf{q}}}\right] \left[\ln \int da' W_{-1}(a,a';t) f(a';t)\right] f(a'';t),$$

where $i\hat{L}_N^{int}$ is a potential part of the Liouville operator,

$$v(a,a';t) = \langle \hat{J}(a,a')\rangle^t_L = \int da'' \mathrm{Sp}(\hat{J}(a) \int\limits_0^1 d\tau (\varrho_q^{kin})^\tau f(a'')(\varrho_q^{kin})^{1-\tau} W_{-1}(a'',a';t) \tag{48}$$

are the generalized flows in collective variables space. Dissipative processes in the transport equations (46), (47) are described by the generalized transport kernels (memory functions):

$$\Phi_{nn}(\mathbf{q},\mathbf{q}',\mathbf{p},\mathbf{p}',a;t,t') = \langle I_n(\mathbf{q},\mathbf{p};t)T_q(t,t')I_n(\mathbf{q},\mathbf{p}';t',\tau)\rangle_L^{t'}, \tag{49}$$

$$\Phi_{n\mathbf{P}}(\mathbf{q},\mathbf{p},\mathbf{k},a;t,t') = \langle I_n(\mathbf{q},\mathbf{p};t)T_q(t,t')I_{\mathbf{P}}(\mathbf{k};t',\tau)\rangle_L^{t'}, \tag{50}$$
$$\Phi_{n\varepsilon}(\mathbf{q},\mathbf{p},\mathbf{k},a;t,t') = \langle I_n(\mathbf{q},\mathbf{p};t)T_q(t,t')I_{\varepsilon}(\mathbf{k};t',\tau)\rangle_L^{t'},$$

$$\Phi_{\mathbf{PP}}(\mathbf{k},a;t,t') = \langle I_{\mathbf{P}}(\mathbf{k};t)T_q(t,t')I_{\mathbf{P}}(\mathbf{k};t',\tau)\rangle_L^{t'}, \tag{51}$$
$$\Phi_{\mathbf{P}\varepsilon}(\mathbf{k},\mathbf{q},a;t,t') = \langle I_{\mathbf{P}}(\mathbf{k};t)T_q(t,t')I_{\varepsilon}(\mathbf{q};t',\tau)\rangle_L^{t'},$$

$$\Phi_{\varepsilon\varepsilon}(\mathbf{k},\mathbf{q},a;t,t') = \langle I_{\varepsilon}(\mathbf{k};t)T_q(t,t')I_{\varepsilon}(\mathbf{q};t',\tau)\rangle_L^{t'}. \tag{52}$$

In particular, the kernel (49) describes the kinetic processes, kernels (50) describe the mutual dynamic correlatios between kinetic and hydrodynamic fluctuations, and (51) describe the dynamic correlation between the viscous and thermal hydrodynamic fluctuations. In expressions (49)–(52) the generalized flows are introduced as follows:

$$\begin{aligned} I_n(\mathbf{q},\mathbf{p};t) &= \Big(1-P(t)\Big)\hat{J}(n_{\mathbf{k}}), \\ I_{\mathbf{P}}(\mathbf{k};t,\tau) &= \Big(1-P(t)\Big)\hat{J}(\mathbf{P}_{\mathbf{k}}), \\ I_{\varepsilon}(\mathbf{k};t',\tau) &= \Big(1-P(t)\Big)\hat{J}(\varepsilon_{\mathbf{k}}), \end{aligned} \tag{53}$$

where $P(t)$ is the Mori projection operator connected to the projection Kawasaki-Gunton operator (33) $P_q(t)a_{\mathbf{k}}\rho_q(t) = \rho_q(t)P(t)a_{\mathbf{k}}$. It is important to note, that the average quantities in (48)–(52) are calculated using the quasiequilibrium statistical operator $\rho_L(a;t)$ (28), therefore the kernels are the functions of collective variables $a_{\mathbf{k}}$.

The obtained system of equations (46), (47) takes into account the effects of time delay and hard nonlocality for variables $\hat{a}$ associated with the contribution of rapid small-scale fluctuations and noncommutative base set $\hat{n}_{\mathbf{q}}(\mathbf{p})$, $\hat{a}_{\mathbf{k}} = \{\hat{n}_{\mathbf{k}},\mathbf{P}_{\mathbf{k}},\hat{\varepsilon}_{\mathbf{k}}\}$. Special difficulty is associated with the structure function $W(a,a';t)$ and averaged velocities $v(a,a';t)$, containing the singular and regular parts:

$$v(a,a';t) = v(a;t)\delta(a-a') + \delta v(a,a';t). \tag{54}$$

If taking into account only the singular parts of functions, we proceed to the local approximation for the statistical operator $\hat{\varrho}(t)$ and the generalized transport equations (49),(50). The

local approximation of generalized Fokker-Planck equation for quantum systems in general case was discussed in paper [26] in detail.

IV. CONCLUSION

We considered two possible variants for consistent description of nonlinear kinetic and hydrodynamic processes for quantum Bose systems, using the Zubarev nonequilibrium statistical operator.

In the first variant, the non-equilibrium Wigner distribution function and the averaged potential energy of interaction were selected as parameters of abbreviated description. Generalized transport equations for them are not closed. In order to close them, the spatial gradients of Lagrange multipliers $\beta_{-\mathbf{q}}(t)$, $\gamma_{-\mathbf{q}}(\mathbf{p};t)$ should be determined from the corresponding self-consistent conditions.

Such calculations, in linear respective to spatial gradients approximation, are complicated and leads to linearized equations of the consistent description of kinetics and hydrodynamics of quantum Bose system [49]. In higher approximations the problems of solving the corresponding nonlinear integral equations would emerge.

In our approach, a part of the average interaction potential energy is connected with the nonequilibrium distribution function of a pair condensate and can be readily extracted. Further, we can obtain the the system of equations for time correlation functions with separation of one-partial and pair-partial Bose condensate both in the Hamiltonian [56–58] and in expressions. Moreover, in our approach we can start from the quasiequilibrium statistical operator (6) to construct a chain of BBGKY equations for nonequilibrium particles distribution functions with the modified boundary conditions (taking into account multipartial correlations), that is similar both for classical [43] and Fermi systems [58]. In the second variant, the non-equilibrium Wigner distribution function and nonequilibrium distribution function of hydrodynamic collective variables (the number density of particles, their momentum and full energy) were selected as abbreviated description parameters. As a result, the system of equations is obtained. The first equation is of the type of kinetic equation for nonequilibrium Wigner distribution function with kinetic transport kernels. The second one is the generalized Fokker-Planck equation for the nonequilibrium distribution function of collective variables taking into account nonmarkov effects. In these equations, the aver-

aging procedure is performed using the quasiequilibrium statistical operator (30), in which the structural function $W(a,a';t)$ (it contains singular and regular parts) is an average of the transition operator $\hat{f}(a)$ from phase variables to collective variables a. Calculation of $W(a,a';t)$ is a key moment here. This is because this function enters the Fokker-Planck equation and is used for calculation of the hydrodynamic speeds $v_l(a,a';t)$ (it contains singular and regular parts) and generalized kernels (49)–(52). In the case studies of nonlinear hydrodynamic fluctuations [59, 60] or the consistent description of kinetic and nonlinear hydrodynamic fluctuations [61, 62] for classical systems, the structural function $W(a;t)$ was calculated by the collective variables method [63]. It opens the opportunity to go beyond the Gaussian approximation for $W(a;t)$ and to propose an approach to calculation of the generalized transport coefficients in higher approximations for the fluctuations, in particular, of the coupled modes type [60, 64]. Calculation of the structure function $W(a,a';t)$ and $v_l(a,a';t)$ in local approximations for quantum Bose systems requires an individual consideration and will be presented in the next paper.

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